MiniTest 4A - MTH 1310

Dr. Graham-Squire, Fall 2012

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(signature)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are <u>not</u> allowed on this test. All cell phones should be turned off and put away, if I see a cell phone out it will be considered an honor code violation.
4. Calculators <u>are</u> allowed on the first 2 questions of the test, however you should still show all of your work. No calculators are allowed on the last 3 questions.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need to use the quadratic formula, it is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. Make sure you sign the pledge.
8. Number of questions = 5. Total Points = 40.

1. (7 points) Lemur populations in captivity are known to follow an exponential growth model. Suppose a zoo starts with 80 lemurs, and it is known that the population will double after 3 years. How many years will it be until they have 300 lemurs? Round your answer to the nearest 0.01 years.

$Q(t) = Q_0 e^{kt}$ ✓ $\quad Q_0 = 80$ ✓

$Q(3) = 160$

$\Rightarrow \frac{160}{80} = \frac{Q(3)}{80} = \frac{80e^{k \cdot 3}}{80}$ ✓

$\Rightarrow 2 = e^{3k}$

$\frac{\ln 2}{3} = k$ ✓

$\Rightarrow Q(t) = 80e^{(\frac{\ln 2}{3})t}$

$300 = 80e^{(\frac{\ln 2}{3})t}$ ✓

$\ln\left(\frac{300}{80}\right) = \left(\frac{\ln 2}{3}\right)t$

$\frac{\ln\left(\frac{300}{80}\right)}{\left(\frac{\ln 2}{3}\right)} = t$

✓

$t \approx 5.72$

2. (5 points) Use logarithmic differentiation to find the $\frac{dy}{dx}$ (or y') for

$$y = (x+3)^{3x}$$

$\Rightarrow \ln y = \ln (x+3)^{3x}$ ✓

$\Rightarrow \ln y = (3x) \cdot \ln(x+3)$ ✓

$\frac{d}{dx}\left(\quad \right)$

$\Rightarrow \frac{y'}{y} = 3 \cdot \ln(x+3) + 3x \cdot \frac{1}{x+3}$ ✓ ✓

$y' = y\left(3 \ln(x+3) + \frac{3x}{x+3} \right)$ ✓

$$\boxed{y' = (x+3)^{3x}\left(3 \ln(x+3) + \frac{3x}{x+3} \right)}$$

-0.5 if no parentheses

Test A

NO CALCULATORS FOR THIS PART

3. (8 points) Find the indefinite integrals:

(a) $\int \left(4e^x - \frac{7}{x}\right) dx$ $= \int (4e^x - 7x^{-1})\,dx$ ✓

$= \boxed{4e^x - 7\ln x + C}$ ✓✓✓

(b) $\int \frac{1+x}{\sqrt{x}}\,dx$

$= \int \left(\frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}}\right) dx = \int \left(x^{-1/2} + x^{1/2}\right) dx$ ✓✓

$= \boxed{2x^{1/2} + \frac{2}{3}x^{3/2} + C}$ ✓ ✓ ✓

0.5

4. (12 points) Calculate the derivative of each function. Simplify your answer if possible.

(a) $f(x) = \left(e^{(x^4-2x)}\right)^3$

$$f'(x) = 3\left(e^{(x^4-2x)}\right)^2 \cdot e^{(x^4-2x)} \cdot (4x^3-2)$$

$$= 3\left(e^{(x^4-2x)}\right)^3 \cdot (4x^3-2)$$

or

$$(12x^3-6)\left(e^{(x^4-2x)}\right)^3$$

(b) $f(x) = \ln(x^4 \cdot e^{x^5})$

$$f(x) = \ln x^4 + \ln e^{x^5}$$

$$f(x) = 4\ln x + x^5$$

$$\boxed{f'(x) = \frac{4}{x} + 5x^4}$$

5. (8 points) Calculate the integral. Make sure to show your work!

$$\int \frac{4x^3 \cdot \ln(x^4+5)}{x^4+5}\,dx$$

$u = \ln(x^4+5)$ ✓

$du = \frac{4x^3}{x^4+5}\,dx$ ✓

$\frac{x^4+5}{4x^3}\,du = dx$ ✓

$$= \int \frac{4x^3}{x^4+5} \cdot (u) \cdot \frac{x^4+5}{4x^3}\,du$$ ✓ ✓

$$= \int u\,du$$ ✓

$$= \frac{1}{2}u^2 + C$$ ✓

$$= \boxed{\frac{1}{2}\left(\ln(x^4+5)\right)^2 + C}$$ ✓

−0.5 if forget +C

Extra Credit (1 point) Calculate the antiderivative: $\int e^{\ln e^3}\,dt$.

$$= \boxed{e^{\ln e^3}\,t + C}$$

or $e^3 \cdot t + C$

MiniTest 4B - MTH 1310

Dr. Graham-Squire, Fall 2012

8:23
8:32
9
⇒ give 40 minutes.

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5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need to use the quadratic formula, it is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. Make sure you sign the pledge.
8. Number of questions = 5. Total Points = 40.

1. (5 points) Use logarithmic differentiation to find the $\frac{dy}{dx}$ (or y') for

$$y = (x-2)^{4x}$$

$\ln y = \ln (x-2)^{4x}$ ✓

$\frac{d}{dx}\left(\ln y = (4x)(\ln(x-2)) \right)$ ✓

$\frac{y'}{y} = 4(\ln(x-2)) + \frac{4x}{x-2}$ ✓✓

$y' = (x-2)^{4x}\left(4\ln(x-2) + \frac{4x}{x-2}\right)$ ✓

-0.5 if no parentheses

2. (7 points) Lemur populations in captivity are known to follow an exponential growth model. Suppose a zoo starts with 70 lemurs, and it is known that the population will double after 5 years. How many years will it be until they have 200 lemurs?

$A = Pe^{rt}$ ✓✓ $P = 70$ ✓ $A = 140$ when $t = 5$

Round answer to nearest 0.01 year.

$\Rightarrow 140 = 70e^{r(5)}$ ✓

$2 = e^{5r}$

$\frac{\ln 2}{5} = r$ ✓

$\Rightarrow 200 = 70e^{\frac{\ln 2}{5} \cdot t}$ ✓

$\frac{200}{70} = e^{\frac{\ln 2}{5} t}$

$\ln\left(\frac{200}{70}\right) = \left(\frac{\ln 2}{5}\right)t$

$\frac{5 \ln\left(\frac{200}{70}\right)}{\ln 2} = t$ ✓

$7.57 \text{ years} = t$

NO CALCULATORS FOR THIS PART

3. (12 points) Calculate the derivative of each function:

(a) $f(x) = \ln\left(x^3 \cdot e^{x^7}\right)$

$f(x) = \ln x^3 + \ln e^{x^7}$

$f(x) = 3\ln x + x^7$

$f'(x) = \frac{3}{x} + 7x^6$

(b) $f(x) = \left(e^{(x^5-3x)}\right)^3$

$f'(x) = 3\left(e^{(x^5-3x)}\right)^2 \left(e^{(x^5-3x)} \cdot (5x^4-3)\right)$

$= 3\left(e^{x^5-3x}\right)^3 (5x^4-3)$

4. (8 points) Find the indefinite integrals:

(a) $\int \frac{1+x}{\sqrt{x}}\,dx$

$= \int \frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}}\,dx$

$= \int (x^{-1/2} + x^{1/2})\,dx$ ✓✓

$= 2x^{1/2} + \frac{2}{3}x^{3/2} + C$

✓ ✓ 0.5

(b) $\int \left(3e^x - \frac{4}{x}\right)dx$ $= \int (3e^x - 4x^{-1})\,dx$ ✓

$= 3e^x - 4\ln x + C$ ✓✓✓

5. (8 points) Calculate the integral. Make sure to show your work!

$$\int \frac{3x^2 \cdot \ln(x^3+5)}{x^3+5}\,dx$$

$u = \ln(x^3+5)$

$du = \frac{3x^2}{x^3+5}\,dx$

$\frac{(x^3+5)\,du}{3x^2} = dx$

$$= \int \frac{3x^2 \cdot u}{x^3+5} \cdot \left(\frac{x^3+5}{3x^2}\,du\right)$$

$$= \int u\,du$$

$$= \frac{1}{2}u^2 + C$$

$$= \boxed{\frac{1}{2}\left(\ln(x^3+5)\right)^2 + C}$$

-0.5 if forget $+C$

Extra Credit (1 point) Calculate the antiderivative: $\int e^{\ln e^2}\,dt$.

$$= e^{\ln e^2} \cdot t + C$$

or $\boxed{e^2 \cdot t + C}$